HAMIKON-AMBALA

CHAPTER 2

DIFFRACTION AND INTERFERENCE

Both Diffraction and Interference Phenomena are a consequence of the wave nature of the light. Whenever a wave is restricted by any obstacle/aperture, diffraction will result. We have interference wherever diffraction patterns from two coherent sources super impose.

Diffraction effects are traditionally classified into either Fresnel or Fraunhofer types. The former concernes with what happens to light in the immediate neighbourhood of a diffracting object - with the object illuminated by a point source at a finite distance. The latter concerns with diffracted light sensed at infinite distances from the diffracting object with the illumination coming usually from an infinite distance point source. Fraun-hofer diffraction is an approximation to the Fresnl case and it is easy to compute..

Fraunhofer diffraction patterns are usually produced by placing any aperture/obstracle in a beam of plane waves and observing the pattern in the back focal plane of a converging lens, which is equivalent to observing at infinity. Using a laser, this can be done by placing the aperture/obstacle in the parallel beam and observing the pattern on the screen either at large distances or at back focal plane of the lens.

The width of the beam can be increased by using a concave lens and a convex lens. Concave lens is used to diverge the beam which is then focussed on a distance screen by a convex lens. The obstacles (apertures, slits, gratings) being used is then placed just beyond the convex lens, so producing a pattern the screen. This arrangement also has the advantage of giving better definition than that obtained without lenses.

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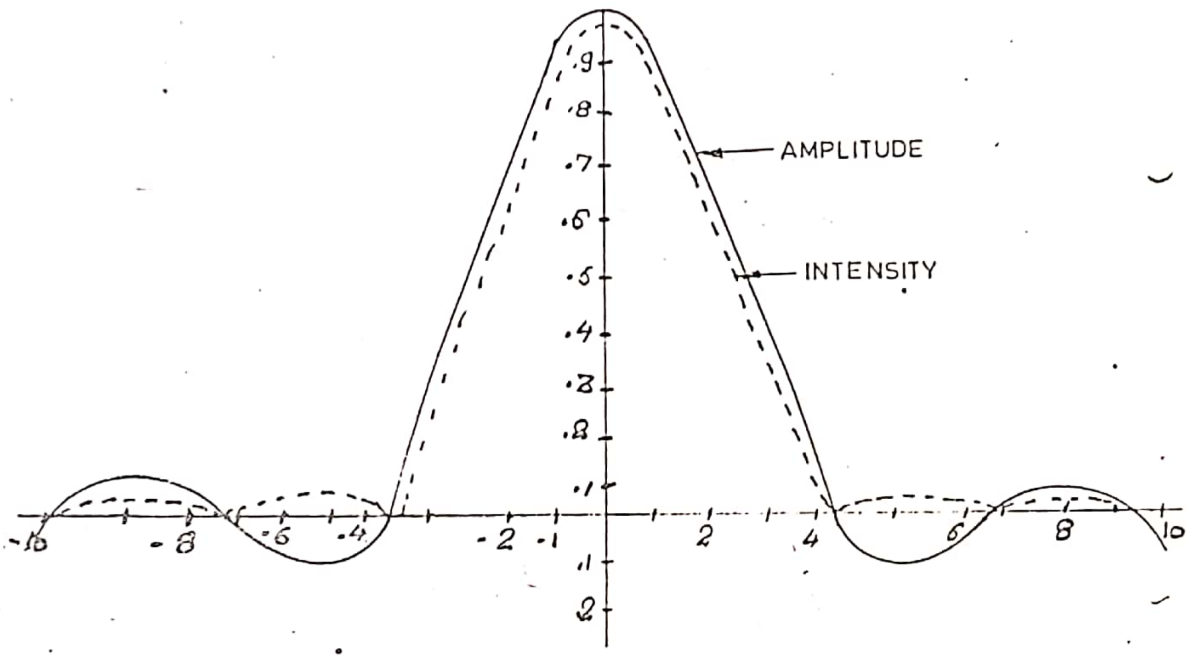

(fig. 2.1 a)

Represents irradiance distribution of a circular aperture.

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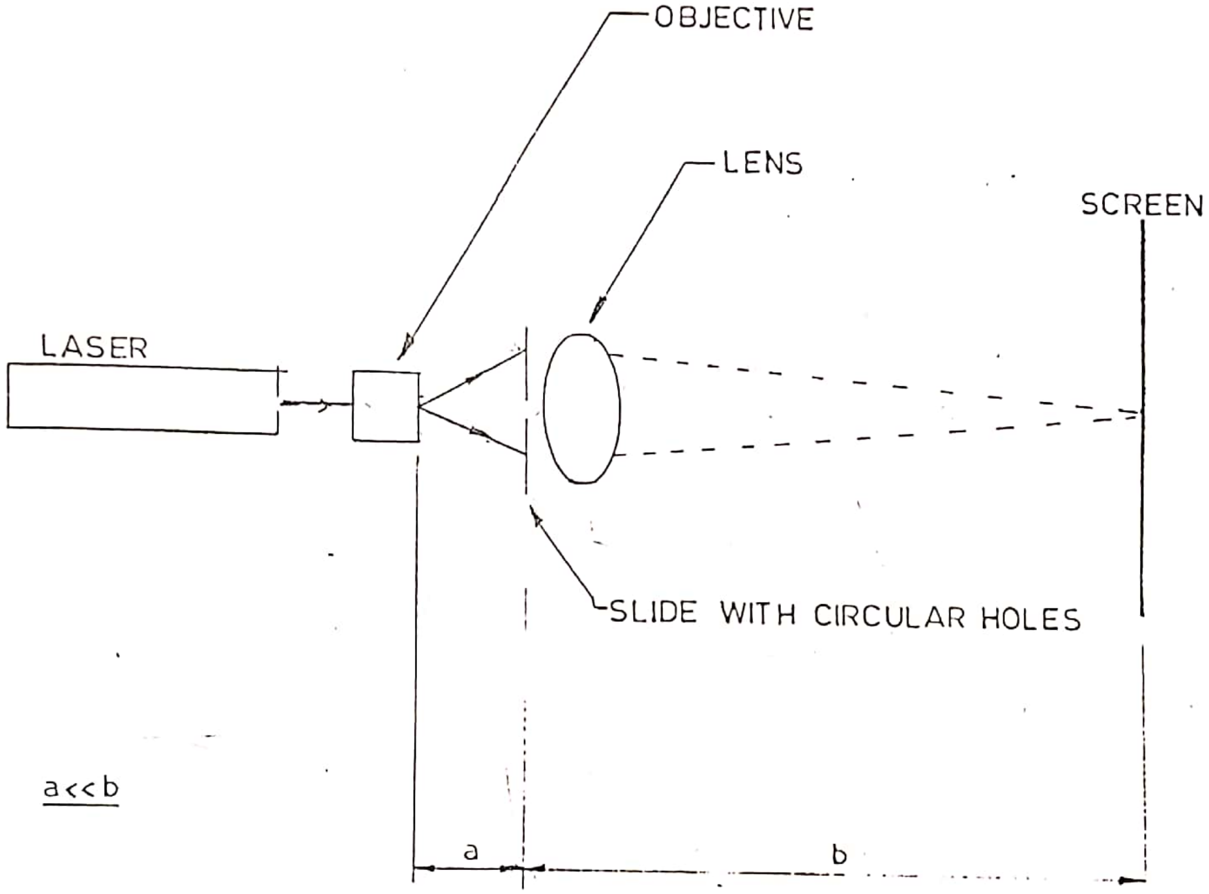


(fig 2.1 b)

Set up for observing Airy's pattern.

To eliminate scattering of light from unwanted surface, a circular stop should be placed between the laser and the diverging lens so that the main beam passes through it but the scattered light does not. This is done by using a opaque screen with a circular hole.

SECTION - I Diffraction Experiments

2.1 FRAUNHOFER DIFFRACTION OF A CIRCULAR APERTURE

The study of this is of considerable importance in optics as most of the optical elements are having circular shape. The imaging performance of the instruments is governed by diffraction of light at the apertures.

The Diffraction pattern resulting from uniformly illuminated circular aperture consists of a central bright region, known as Airy's Disc, surrounded by a number of fainter rings. Each ring is separated by a circle of zero intensity. The irradiance distribution in this pattern can be described by

$$I = Io \left(\frac{2 J_1(x)}{x} \right)^2 \quad \text{where}$$

Io = peak irradiance in image

$$J_1(x) = X \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-2}}{(n-1)!\; n!\; 2^{2n-1}}$$

$J_1(x)$ = Bessel function of the first kind of order unity and

$$x = \frac{\pi D \sin \theta}{\lambda} \quad \text{where}$$

λ = Wavelength of light

D = Aperture diameter and

θ = Angular radius from pattern maximum

Fig. (2.1 a) shows the irradiance distribution of circular aperture.

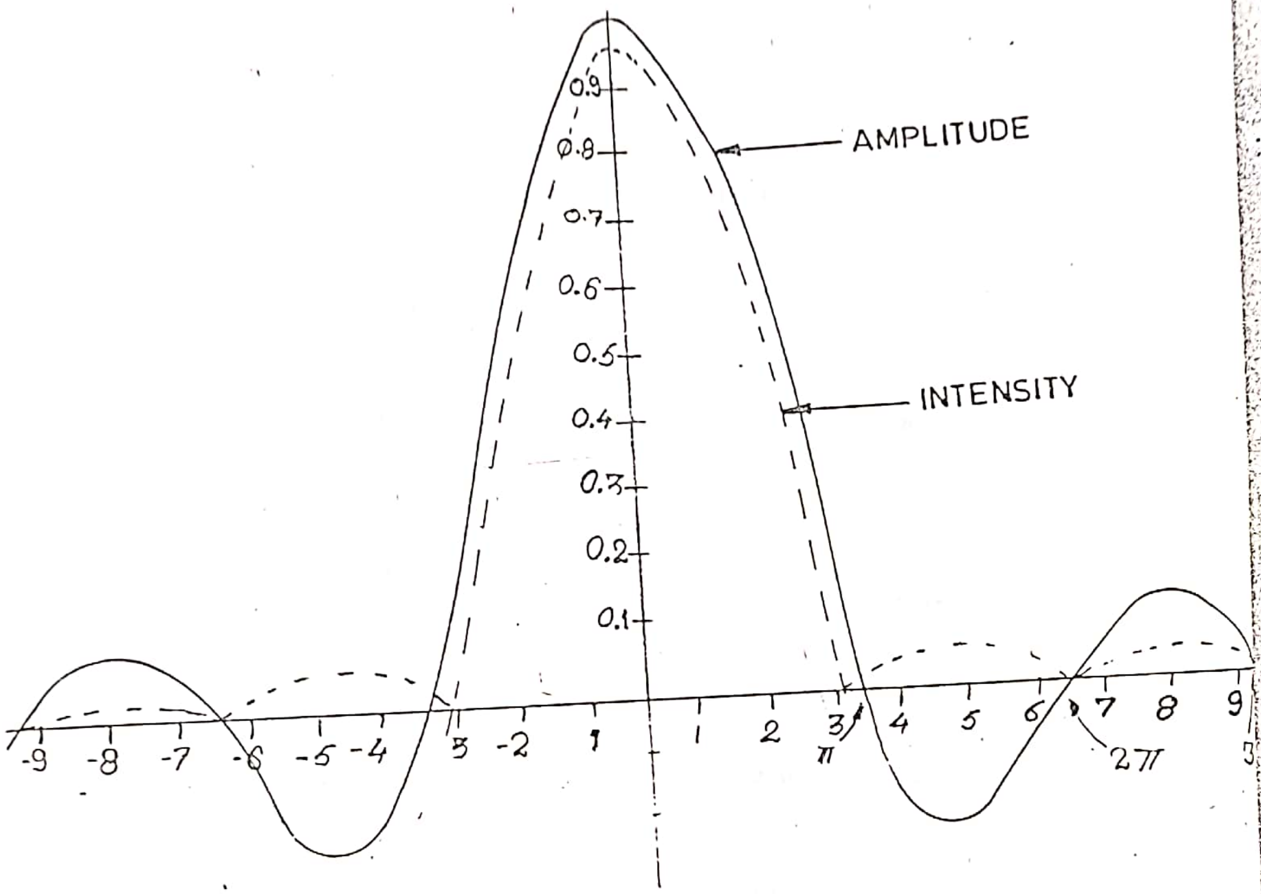
The width of the central maxima is approximately double that of the secondary maxima. The minima occur at the zero's of the Bessel function $J_1(x)$. The first minimum occur when

$$x = 1.22\pi$$

Then $$\sin\theta = \frac{1.22\lambda}{D}$$

where $\sin\theta = \frac{\rho}{z}$ is the sine of the angle which the line from the center of the aperture to any point on the observation plane makes with the optic axis, ρ is the linear distance of the point from the axis, z is the distance between the aperture and the observation plane.

Fig. (2.1 b) shows the set up for observing the patterns.



(fig 2.2)

Shows the irradiance distribution of a single slit.

2.2 Fraunhofer Diffraction By Single Slit.

This is one of the simple case of diffraction. Assuming the slit to be infinitely long in one direction, the irradiance distribution at point P is given by

$$I = I_0 \left[\frac{\sin(ka \sin\alpha)}{ka \sin\alpha} \right]^2 \quad \text{where}$$

I_0 is the irradiance at $\alpha = 0$

2a is the slit width

α is the angle of the diffracted ray making with the axis.

The minimum occures when

$$2a \sin\alpha = n\lambda$$

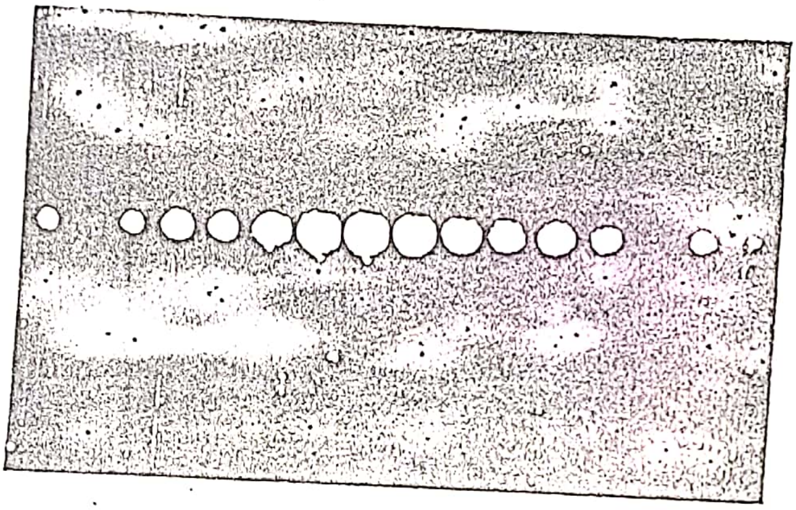
i.e. when $\alpha = \sin^{-1}\left(\frac{\lambda}{2a}\right)$, $\sin^{-1}\left(\frac{2\lambda}{2a}\right)$ etc.

The condition n =o gives the maximum irradiance Io that occurs in the direction α = o. The angular separation between the consecutive minima is $\frac{\lambda}{2a}$. The exception being the first order minima about the central maximum which are λ/a apart. The central maximum has twice the width of the secondary maxima, which come closer as the slit width is increased. Fig. (2.2) shows the irradiance distribution.

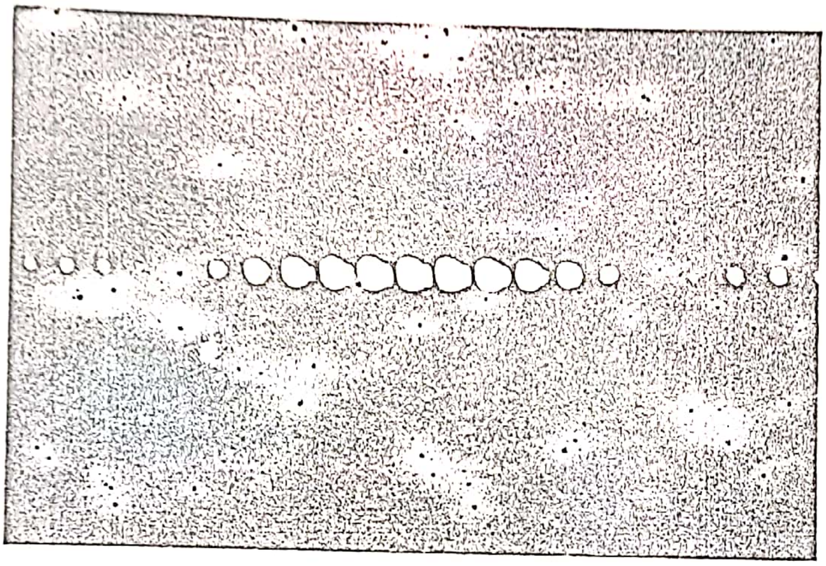
Thus, if slit width 2a is known λ can be determined and vice-versa.

The experiment can be also done a using a adjustable slit and the variation of the pattern with increasing, slit width can also be seen. (One can easily see the broadening of the pattern as slit is narrowed.)

Diffraction pattern will became brighter in case the experiment is performed without using lens as now more energy passes through the slit.



MULTIPLE SLIT-3

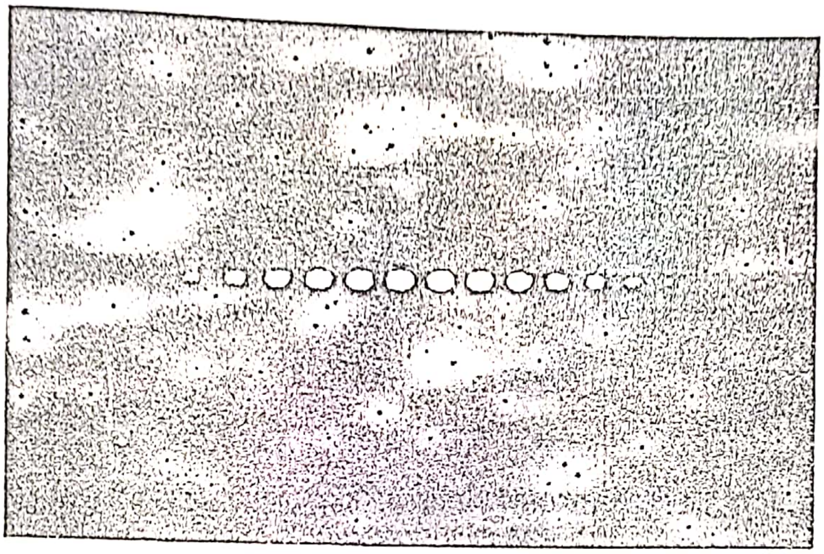


MULTIPLE SLIT-5

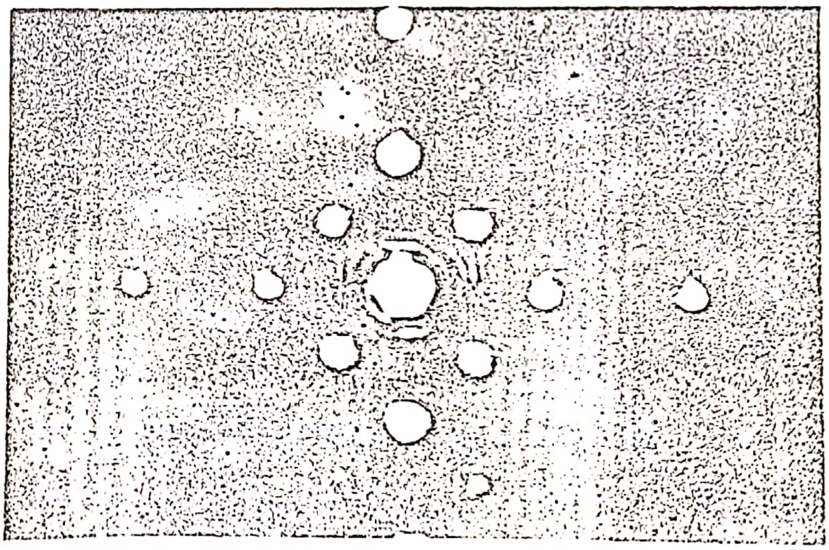
2.3 Diffraction by multiple slits

As the number of slits increases, the brightness of the pattern increases, but the broad central patch within which the fringes lie does not change from its size for 1 slit, but fainter patches on either side of this central region becomes more visible as the number of slits increases. The width of the central region depends on slit width.

As the number of slits increases, the width of the bright fringes decreases but separation of points of greatest brightness within the pattern remains fixed. Secondary maxima equal to number of slits-2, appear in between principal maxima. The separation of these points depends on slit separation, their narrowness depends on how many slits are used.



COARSE GRATING-DIFFRACTION PATTERN



MESH DIFFRACTION PATTERN

2.4 Diffraction by gratings.

Consider one dimensional array of N slits with the intercenter separation of d. If the aperture is a slit of width 2b, then the irradiance distribution at the focal plane of lens due to diffraction at the grating is given by

$$I(p) = I_o \left(\frac{\sin kpb}{kpb}\right)^2 \left(\frac{\sin N\delta/2}{\sin \delta/2}\right)^2$$

where I_o is the irradiance in the direction $\theta = 0$

δ is the phase difference $= kd \sin\theta$

The first term in the above equation represents the irradiance distribution due to a single slit and the second term represents the interference due to N slits. This modulates the diffraction pattern of a single slit. The fringes produced due to interference are narrow, compared to single slit diffraction pattern. Number of Principal maxima lie in central maximum and are given by

$$d \sin\theta = m\lambda \qquad \text{------------------} \quad (2)$$

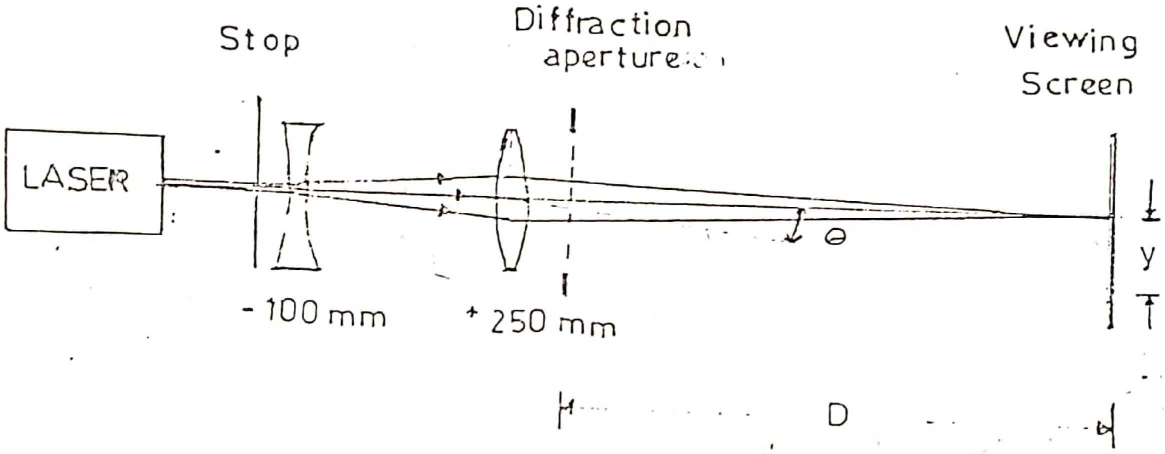
$$m = 0, \pm 1, \pm 2 \text{ ----------------------}$$

The different integral values of is gives different orders of the spectrum, m = 1 is first order, m = 2 second order and so on. There are (N-1) minima and (N-2) secondary maxima between any two consecutive principal maxima. As N increases, the magnitude of principal maxima increases and that of the secondary maxima decreases.

Equation (2) is known as grating equation for normal incidence of light. In general, when the beam is incident at an angle i, then the grating equation is given by

$$d(\sin i - \sin\theta) = m\lambda$$

For normal incidence, if m, θ and d (grating constant) are known, can be determined. If a lens of focal length f is used to form the diffracted



(fig. 2.3)

Shows the arangement for observing Fraunhofer diffraction patterns of different apertures.

beam, then the mth order distance from the optical axis is given as, provided $x_m << f$

$$\sin\theta = \frac{x_m}{(x_m^2 + f^2)^{\frac{1}{2}}} = \frac{d}{m}\frac{x_m}{(x_m^2 + f^2)^{\frac{1}{2}}}$$

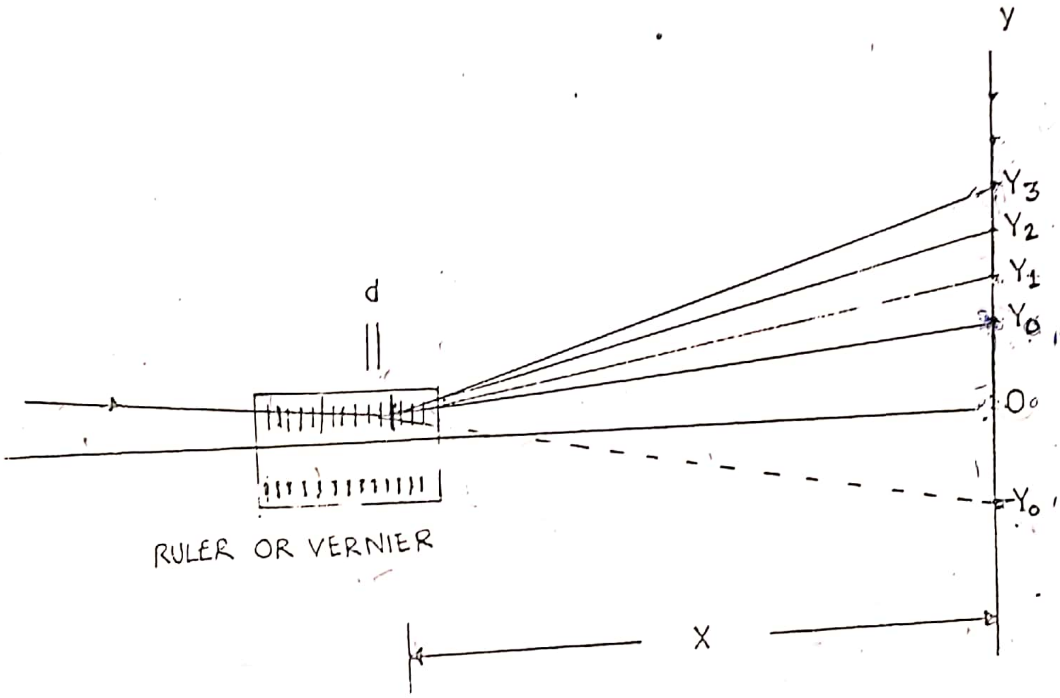
If λ is known, then the above equation can be used to determine grating constant.

GENERAL PROCEDURE (§ 2.1, 2.2, 2.3 and 2.4).

Fig.(2.3) shows the set up for observing the diffraction patterns of the following apertures:

1. Circular aperture.
2. Single slit or an adjustable slit.
3. Multiple slits.
4. Diffraction gratings.
5. Mesh.

1. Switch on the laser and direct the beam metres towards a viewing screen situated about 3-5 metres away. The laser is held in the holder, properly adjusted.
2. Mount a 10 cm diverging lens in the adjustable lens holder about a few cms from the laser and then focus the light on the screen to a spot with the 25 cm converging lens. The light should pass through the centre of the lenses. Adjust the distance so that we have fairly uniform size of the spot.
3. A stop may be mounted alongwith the diverging lens to reduce the amount of scattered light falling on the screen.
4. Insert the aperture whose diffraction pattern is desired in the beam as shown in the set up. We will get Fraunhofer diffraction pattern.
5. For making any measurement in the pattern, mark with a pencil on a graph paper pasted on the screen. Switch off the laser and then make required measurement.



(fig 2.4)

Diffraction pattern of rulings of a vernier.

2.5 Measuring the wavelength of Laser Light using Vernier Caliper.

A polished steel rule such as vernier caliper can be used as a reflection grating. When light strikes at grazing incidence, light is diffracted into many orders depending on the spacing and accuracy of the graduations. This was first demonstrated by A.L.Schawlow using a steel (or plastic) scale. The wavelength of the light is obtained by measuring the pattern spacings and the distance from the vernier to the screen. This is given by

$$\lambda = \frac{d}{2n} \times \left(\frac{y_n^2 - y_o^2}{x^2}\right) \quad \text{where}$$

d is the spacing between rulings; n is integer (the diffraction order) y_n and y_o are measured from O along the projection screen.

See Fig. (2.4)

The intersection of the plane of the grating with the screen (O) lies halfway between the sports of the direct beam (-Yo) and the Zero order diffracted beam which is specularly reflected (Yo); X is the distance between the vernier and the screen.

PROCEDURE

(1) Switch on the laser and adjust the levelling screws of the holder in such a way that the laser tube is very much titled.

(2) The vernier caliper is placed on the rotatable Mount. Adjust the height of the platform so that laser light falls at a grazing angle (i=87°). This can be done using the three levelling screws provided at the bottom of the platform. The diffraction pattern is observed at a distance of 3-4 metres away from the caliper.

(3) Mark the position of various orders and direct spot without any diffraction on the screen.

(4) Switch off the laser and note the distances of the spots from the inter section point O as shown in the figure. Note also the distance between the screen (a mm-graph paper pasted on the screen) and the caliper calculate the wavelength of light using the above formula.